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Inverse filter criteria for blind deconvolution and equalization using two cumulants

Chong-Yung Chi*, Mei-Chyn Wu

Department of Electrical Engineering, National Tsing Hua University, Hsinchu, Taiwan, Republic of China

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Abstract

Cumulant (higher-order statistics) based inverse filter criteria maximizing $J_{r,m} = |C_m|^r/|C_r|^m$, where $m \neq r$ and C_m (C_r) denotes the mth-order (rth-order) cumulant of the inverse filter output, have been proposed for blind deconvolution and equalization with only non-Gaussian output measurements of an unknown linear time-invariant (LTI) system. This paper shows that the maximum of $J_{r,m}$, associated with the true inverse filter of the unknown LTI system, exists only for r to be even and $m > r$, otherwise $J_{r,m}$ is unbounded. The admissible values for $(r, m) = (2s, l + s)$ where $l > s \geqslant 1$ include $(2, 3)$, $(2, 4)$ and $(4, 6)$ proposed by Tugnait, Wiggins, Shalvi and Weinstein in addition to the new ones such as $(2, 5)$, $(2, 6)$ and $(4, 5)$. Some simulation results associated with the inverse filter criteria $J_{r,m}$ with the admissible values for (r, m) are then provided. Finally, we draw some conclusions.

Zusammenfassung

Zur blinden Entfaltung und Entzerrung unter nicht-gaußschen Empfangssignalen bei unbekannten linearen zeitinvarianten Systemen (LTI) wurden Kumulanten-basierte (higher order statistics) Kriterien für inverse Filter vorgeschlagen, die $J_{r,m} = |C_m|^r/|C_r|^m$ maximieren, wobei $m \neq r$ und C_m (C_r) Kumulanten m-ter (r-ter) Ordnung des Ausgangssignals des inversen Filters bezeichnen. Diese Arbeit zeigt, daß das Maximum von $J_{r,m}$, verbunden mit dem wahren inversen Filter des unbekannten LTI-Systems, nur für gerade r und $m > r$ existiert, andernfalls ist $J_{r,m}$ unbegrenzt. Die zulässigen Werte für $(r, m) = (2s, l + s)$, mit $l > s \geqslant 1$, enthalten $(2, 3)$, $(2, 4)$ und $(4, 6)$ wie von Tugnait, Wiggins, Shalvi und Weinstein vorgeschlagen und weiterhin einige neue Werte wie $(2, 5)$, $(2, 6)$ und $(4, 5)$. Es werden einige Simulationsergebnisse verbunden mit den inversen Filter Kriterien $J_{r,m}$ mit zulässigen Werten für (r, m) wiedergegeben. Die Arbeit schließt mit eine Konklusion.

Résumé

Des critères de filtre inverse basés sur les cumulants (statistiques d'ordre supérieur) maximisant $J_{r,m} = |C(m)|^r/|C(r)|^m$, ou $m \neq r$ et C_m (C_r) denote le cumulant de m-ième ordre (r-ième ordre) de la sortie du filtre inverse, ont été proposées pour la déconvolution et l'égalisation aveugles, connaissant seulement des mesures non gaussiennes de la sortie d'un système linéaire invariant dans le temps (LIT) inconnu. Cet article montre que le maximum de $J_{r,m}$, associé au vrai filtre inverse du système LIT inconnu, existe seulement pour r pair et $m > r$, sinon $J_{r,m}$ est non limité. Les valeurs admissibles pour $(r, m) = (2s, l + s)$ ou $l > s \geqslant 1$ incluent les couples $(2, 3)$, $(2, 4)$ et $(4, 6)$ proposés par Tugnait, Wiggins, Shalvi et Weinstein

*Corresponding author. Tel.: 886-35-731156. Fax: 886-35-715971. E-mail: cychi@ee.nthu.edu.tw.

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ainsi que d'autre plus récents, tels (2, 5), (2, 6) et (4, 5). Des résultats de simulation associées aux critères de filtre inverse $J(m,r)$ avec les valeurs admissibles pour (m,r) sont ensuite donnés. Enfin, nous esquissons quelques conclusions.

Keywords: Inverse filter criteria; Blind deconvolution; Equalization; Cumulant

1. Introduction

Blind deconvolution as well as equalization is a quite known statistical signal processing problem to estimate the desired signal $u(n)$ only with a given set of measurements $x(n)$, $n = 0, 1, \ldots, N-1$, based on the following convolutional model:

$$x(n) = u(n) * h(n) + w(n) = \sum_{i=-\infty}^{\infty} h(i)u(n-i) + w(n), \tag{1}$$

where $w(n)$ is measurement noise and $h(n)$ is an unknown linear time-invariant (LTI) system which corresponds to such as the source wavelet in seismic deconvolution, the channel impulse response in channel equalization and the vocal-tract filter in speech processing. A major conventional approach to this problem is the correlation (second-order statistics) based predictive deconvolution [11, 17]. The deconvolved signal is obtained by processing $x(n)$ with a minimum-phase linear prediction error (LPE) filter [9, 11, 17], which corresponds to an estimate for the inverse filter of $h(n)$ except for phase distortion because $h(n)$ may not be minimum-phase in practice. In other words, LPE filters are blind to the phase of $h(n)$. Moreover, LPE filters are sensitive to additive noise since correlations of $x(n)$ contain noise correlations in addition to correlations of the noise free signal $u(n)*h(n)$. Recently, higher-order ($\geqslant 3$) statistics (HOS) [14, 15], known as cumulants, have been considered in various signal processing areas where $x(n)$ is non-Gaussian and measurement noise $w(n)$ is Gaussian with unknown statistics, partly because cumulants of $x(n)$ can be used to extract not only the amplitude information but also phase information of $h(n)$ and partly because higher-order cumulants of Gaussian noise $w(n)$ are zero.

Assume that $v(n)$ is an estimate for the inverse filter of $h(n)$ and that $e(n)$ is the output of the filter $v(n)$ in response to noisy measurements $x(n)$, i.e.,

$$e(n) = x(n) * v(n). \tag{2}$$

The goal of cumulant based inverse filter criteria [1–7, 16, 18–20] is to find the optimum $v(n)$ from higher-order cumulants of $e(n)$ such that $v(n)*h(n)$ approximates $\alpha\delta(n-\tau)$ where α is a nonzero scale factor and τ is an unknown time delay. Thus, the associated inverse filter output signal $e(n) \approx \alpha u(n-\tau)$ provides sufficient information about the driving input of the unknown LTI system $h(n)$. As mentioned above, when $v(n)$ is an LPE filter and $h(n)$ is nonminimum-phase, the predictive deconvolved signal $e(n) \approx \alpha u(n) * h'(n)$ for this case, where $h'(n)$ is an all-pass system, no longer provides accurate information about $u(n)$ even when the signal-to-noise ratio (SNR) is quite high.

A class of cumulant based inverse filter criteria [16, 18–20] finds the optimum inverse filter $v(n)$ by maximizing an objective function with the following form:

$$J_{r,m}(v(n)) = \frac{|C_m|^r}{|C_r|^m}, \tag{3}$$

where $m \neq r$, $m \geqslant 2$, $r \geqslant 2$ and C_m (C_r) is the mth-order (rth-order) cumulant of $e(n)$, i.e.,

$$C_m = \mathrm{Cum}[x_1 = e(n),\ x_2 = e(n), \ldots, x_m = e(n)], \tag{4}$$

where $\mathrm{Cum}[x_1, x_2, \ldots, x_m]$ denotes the joint cumulant of random variables $x_1, x_2, \ldots, x_m$ [14, 15]. Note that $J_{r,m}$ uses only two cumulants with different orders. For instance, Wiggins [20] proposed an inverse filter criterion by maximizing $J = E[e^4(n)]/(E[e^2(n)])^2$, which is related to $J_{r,m}$ by $|J-3|^2 = J_{2,4}$. Shalvi and Weinstein [16] proposed an inverse filter criterion by maximizing $|C_4|$ subject to the constraint $E[e^2(n)] = E[u^2(n)]$. Tugnait [18, 19] also proposed inverse filter criteria by maximizing $J_{2,3}$ or $J_{2,4}$, or $J_{4,6}$. However, for other choices of r and m, it is still unknown whether maximizing $J_{r,m}$ can lead to the inverse filter of $h(n)$. Chen and Chi [1] and Chen et al. [4] proposed some inverse filter criteria using a slice of cumulants of $e(n)$. Chi and Kung [6,8] estimated the inverse filter $v(n)$ by maximizing a single cumulant $|C_m|$ for $m \geqslant 3$ when $h(n)$ is an all-pass system.

In this paper, we show that maximizing the objective function $J_{r,m}$ given by (3) is applicable only for some certain choices of r and m which lead to new inverse filter criteria in addition to the aforementioned existing inverse filter criteria as special cases of the admissible r and m. Section 2 presents the admissible values for cumulant orders r and m required by the inverse filter criteria $J_{r,m}$. Then some simulation results are provided in Section 3. Finally, we draw some conclusions.

2. Admissible cumulant orders for the inverse filter criteria $J_{r,m}$

Assume that $x(0), x(1), \ldots, x(N-1)$ are given noisy measurements generated from the convolutional model given by (1) with the following assumptions:

(A1) The unknown LTI system $h(n)$ is causal stable with either minimum phase or nonminimum phase and a stable inverse filter $h_{\text{I}}(n)$ of $h(n)$ exists.

(A2) The driving input $u(n)$ is real, zero-mean, stationary, independent identically distributed (i.i.d.) non-Gaussian with variance σ_u^2 and mth-order cumulant γ_m where $m \geqslant 3$.

(A3) Measurement noise $w(n)$ is Gaussian with unknown statistics.

(A4) The input $u(n)$ and the noise $w(n)$ are statistically independent.

Assume that the inverse filter estimate $v(n)$ for $h_{\text{I}}(n)$ is a stable filter. Then the inverse filter output $e(n)$ given by (2) can be expressed as

$$e(n) = u(n) * g(n) + w'(n), \tag{5}$$

where $w'(n) = w(n) * v(n)$ is also a Gaussian noise sequence since $w(n)$ is Gaussian by (A3) and $g(n)$ is also a stable filter given by

$$g(n) = h(n) * v(n). \tag{6}$$

The admissible cumulant orders for the inverse filter criteria given by (3) are described in the following theorem.

Theorem 1. *Assume that $x(n)$ is the noisy signal generated from the model given by* (1) *under the previous assumptions* (A1)–(A4). *Then the following two statements are true*:

(S1) *$J_{r,m}(v(n))$ is unbounded except for the case that $r = 2s$ (i.e., r is even), $m = l + s > r$ where $l > s \geqslant 1$. Moreover,*

$$\max\{J_{2s,l+s}(v(n))\} = \frac{|\gamma_{l+s}|^{2s}}{|\gamma_{2s}|^{l+s}}. \tag{7}$$

(S2) *The optimum $\hat{g}(n)$ associated with $J_{2s,l+s}(\hat{v}(n)) = \max\{J_{2s,l+s}(v(n))\}$ where $l > s \geqslant 1$ is given by*

$$\hat{g}(n) = \alpha\delta(n - \tau), \tag{8}$$

where $\alpha \neq 0$ is a scale factor and τ is an unknown integer, for the two cases that $s = 1$ with $SNR = \infty$ and $s > 1$ with finite SNR.

The proof for Theorem 1 is based on the well-known Cauchy–Schwarz inequality

$$\left\{\sum_{n=-\infty}^{\infty} a(n)b(n)\right\}^2 \leqslant \left\{\sum_{n=-\infty}^{\infty} a^2(n)\right\}\left\{\sum_{n=-\infty}^{\infty} b^2(n)\right\} \tag{9}$$

and the following lemma.

Lemma 1. *Let $a(n)$ be a nonzero sequence with finite l_p norm $\{\sum_{n=-\infty}^{\infty}|a(n)|^p\}^{1/p}$, then*

$$\left\{\sum_{n=-\infty}^{\infty} |a(n)|^l\right\}^{1/l} \leqslant \left\{\sum_{n=-\infty}^{\infty} |a(n)|^s\right\}^{1/s} \tag{10}$$

where l and s are positive integers and $l > s$.

The proof of Lemma 1 and that of Theorem 1 are given in Appendices A and B, respectively.

Some worthy remarks for the inverse filter criteria $J_{2s,l+s}$ are summarized as follows:

(R1) For $(s,l) = (1,2)$, $(s,l) = (1,3)$, and $(s,l) = (2,4)$, $J_{2s,l+s}$ reduces to Tugnait's inverse filter criteria $J_{2,3}$, $J_{2,4}$, and $J_{4,6}$, respectively. For any other choices of (s,l), $J_{2s,l+s}$ such as $J_{2,5}$, $J_{2,6}$ and $J_{4,5}$ are new.

(R2) In practice, the two cumulants C_{2s} and C_{l+s} of $e(n)$ required by $J_{2s,l+s}$ must be replaced by the corresponding sample cumulants $\hat{C}_{2s}$ and $\hat{C}_{l+s}$ calculated from $e(n)$, $n = 0, 1, \dots, N-1$. However, $\hat{C}_{2s}$ and $\hat{C}_{l+s}$ are known to be consistent estimates [15] for C_{2s} and C_{l+s}, respectively. Therefore, the optimum estimate $\hat{v}(n)$ is also a consistent estimate for $h_{\mathrm{I}}(n)$ except for a scale factor and an unknown time delay.

Next, let us present how we find the optimum inverse filter $\hat{v}(n)$ associated with the inverse filter criteria $J_{2s,l+s}$ with finite data set $\{x(0), x(1), \dots, x(N-1)\}$. The inverse filter $v(n)$ is assumed to be a causal FIR filter of order equal to L. Then the inverse filter output $e(n)$ given by (2) can be expressed as

$$e(n) = \boldsymbol{v}^{\mathrm{T}} \boldsymbol{x}_n, \tag{11}$$

where $\boldsymbol{v}$ and $\boldsymbol{x}_n$ are $(L+1) \times 1$ column vectors given by

$$\boldsymbol{v} = [v(0),\ v(1),\ \dots, v(L)]^{\mathrm{T}} \tag{12}$$

and

$$\boldsymbol{x}_n = [x(n), x(n-1), \dots, x(n-L)]^{\mathrm{T}}, \tag{13}$$

respectively. It can be easily seen that the inverse filter criteria $J_{2s,l+s}$ where $l > s \geqslant 1$ are highly nonlinear functions of $\boldsymbol{v}$ since sample cumulants $\hat{C}_{2s}$ and $\hat{C}_{l+s}$ are nonlinear functions of $\boldsymbol{v}$. Thus, one has to resort to iterative numerical optimization algorithms for finding the optimum $\boldsymbol{v}$. A gradient type numerical optimization algorithm is used to search for the optimum inverse filter estimate $\hat{\boldsymbol{v}}$. At the ith iteration, $\hat{\boldsymbol{v}}_i$ is updated with

$$\hat{\boldsymbol{v}}_i = \hat{\boldsymbol{v}}_{i-1} + \rho \boldsymbol{g}_{i-1}, \tag{14}$$

where ρ is a positive constant and $\boldsymbol{g}_{i-1}$ is the gradient of $J_{2s,l+s}$ with respect to $\boldsymbol{v}$ for $\boldsymbol{v} = \hat{\boldsymbol{v}}_{i-1}$. The detailed calculation for $\boldsymbol{g}_{i-1}$ involves only lengthy and tedious algebraic manipulations which are therefore omitted here. However, when updating $\hat{\boldsymbol{v}}_i$ by (14) results in $J_{2s,l+s}(\hat{\boldsymbol{v}}_i) < J_{2s,l+s}(\hat{\boldsymbol{v}}_{i-1})$, one can continually decrease ρ by $\rho/2$ until $J_{2s,l+s}(\hat{\boldsymbol{v}}_i) > J_{2s,l+s}(\hat{\boldsymbol{v}}_{i-1})$. As other numerical optimization algorithms, an initial condition for $\hat{\boldsymbol{v}}_0$ is needed to initialize the above numerical optimization algorithm. For instance, a minimum-phase LPE filter can be used as the initial condition for $\hat{\boldsymbol{v}}_0$. Next, let us present some simulation results to justify that the inverse filter criteria $J_{2s,l+s}$ where $l > s \geqslant 1$ can be used for the estimation of the inverse filter of the unknown LTI system $h(n)$ and for deconvolution.

3. Simulation results

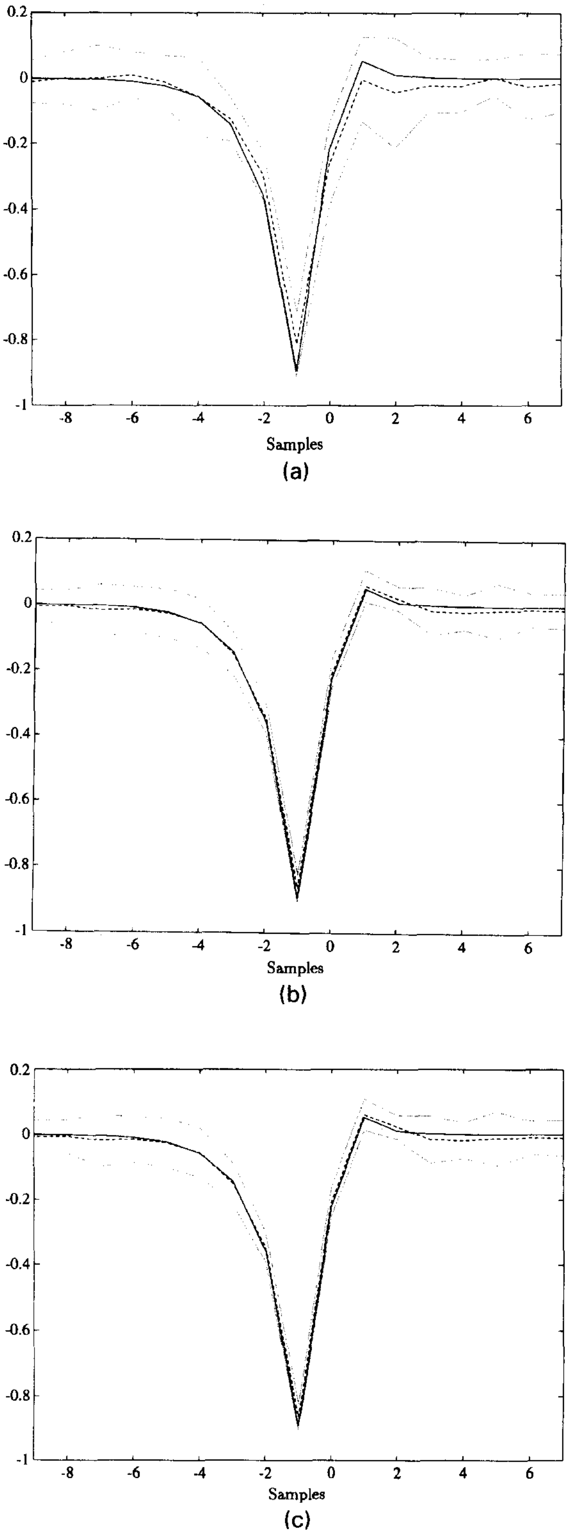
In this section, two simulation examples are to be presented to support the proposed unified class of inverse filter criteria $J_{2s,l+s}$ where $l > s \geqslant 1$ presented in Theorem 1. The first example is a performance test to the new inverse filter criterion $J_{2,5}$ (i.e., $s = 1$ and $l = 4$ in $J_{2s,l+s}$). The second example is seismic deconvolution using the new inverse filter criterion $J_{2,6}$ (i.e., $s = 1$ an $l = 5$ in $J_{2s,l+s}$).

Example 1 (*Performance test*). In this example, the driving input $u(n)$ used was a zero-mean, i.i.d. Exponential random sequence with variance $\sigma_u^2 = 1$, skewness $\gamma_3 = 2$, kurtosis $\gamma_4 = 6$ and fifth-order cumulant $\gamma_5 = 24$. The unknown LTI system $h(n)$ used was a nonminimum-phase second-order autoregressive moving average (ARMA) system with the transfer function (taken from [7]) given by

$$H(z) = \frac{1 - 2.7z^{-1} + 0.5z^{-2}}{1 + 0.1z^{-1} - 0.12z^{-2}}. \tag{15}$$

Synthetic noisy data $x(n)$ of two different data lengths ($N = 2048$ and 4096, respectively) were generated from (1) for two different SNRs (10 dB and 20 dB) and noise $w(n)$ being white Gaussian. The objective function $J_{r,m}$ given by (3) with $(r, m) = (2, 5)$ was used to estimate the inverse filter $v(n)$ which was assumed to be a causal FIR filter of order $L = 16$. An initial condition $\boldsymbol{v} = [0, \dots, 0, 1, v_{\mathrm{b}}(1), \dots, v_{\mathrm{b}}(8)]^{\mathrm{T}}$ was used to initialize the associated gradient type optimization algorithm for searching for the optimum $\hat{\boldsymbol{v}}$ where $\{1, v_{\mathrm{b}}(1), \dots, v_{\mathrm{b}}(8)\}$ were the coefficients of an eighth-order LPE filter obtained by the well-known Burg's algorithm [9].

The simulation results over 30 independent runs associated with $J_{2,5}$ for ($N = 2048$, SNR = 10 dB), ($N = 4096$, SNR = 10 dB) and ($N = 4096$, SNR = 20 dB) are shown in Fig. 1. Note that each inverse



filter estimate $\hat{v}(n)$ obtained from each independent run was normalized by $\|\hat{\mathbf{v}}\| = 1$ and the associated unknown time delay was artificially removed. From Fig. 1, one can see that the estimated inverse filter is basically unbiased. Note that the estimated inverse filter has a larger variance for $N = 2048$ than for $N = 4096$ (see Figs. 1(a) and (b)) and that the variance of the estimated inverse filter for SNR = 10 dB is similar to that for SNR = 20 dB (see Figs. 1(b) and (c)) for this case. Nevertheless, these simulation results justify that $J_{2s,l+s}$ where $l > s \geqslant 1$ can be used to estimate the inverse filter of an unknown nonminimum-phase LTI system.

Because $\gamma_3 = 2 \neq 0$, we also performed the same performance test to Tugnait's criterion $J_{2,3}$ with the same simulation data. Simulation results show that the performance of $J_{2,3}$ is superior to that of the new criterion $J_{2,5}$ for this example. With no doubt, when $J_{2,3}$ is sufficient in certain practical applications, $J_{r,m}$ for higher admissible r and m such as $J_{2,5}$ are redundant. Nevertheless, we would like to emphasize that when cumulants of measurements are small or close to zero for lower r and m such as $J_{2,3}$ and $J_{2,4}$, the inverse filter criteria $J_{r,m}$ only for higher admissible r and m such as $J_{2,5}$ can be considered for the estimation of inverse filter.

Example 2 (*Seismic deconvolution*). In seismic deconvolution, a source wavelet $h(n)$ which is causal but not necessarily minimum-phase is input to the Earth and the received noisy data can be modeled as (1) where $u(n)$ is a reflectivity sequence of the local geology and $w(n)$ is measurement noise. Because the reflectivity sequence $u(n)$ is generally a non-Gaussian sparse spike train with random amplitudes, Kormylo and Mendel [10, 12, 13] proposed a Bernoulli–Gaussian (B–G) model for a sparse reflectivity sequence as follows:

$$u(n) = r(n) \cdot q(n), \tag{16}$$

Fig. 1. Simulation results associated with $J_{2,5}$ for Example 1. The average (dashed line) as well as $\pm$ one standard deviation (dotted lines) of 30 independent inverse filter estimates together with the true inverse filter (solid line) for (a) $N = 2048$ and SNR = 10 dB, (b) $N = 4096$ and SNR = 10 dB, and (c) $N = 4096$ and SNR = 20 dB, respectively.

where $r(n)$ is a zero-mean white Gaussian random process with variance σ_r^2 and $q(n)$ is a Bernoulli sequence for which

$$P_r[q(n)] = \begin{cases} \lambda, & q(n) = 1, \\ 1-\lambda, & q(n) = 0. \end{cases} \tag{17}$$

In this example, a B–G sequence with $\sigma_r^2 = 1$ and $\lambda = 0.1$ (taken from [5, 6, 8]) and a third-order nonminimum-phase wavelet (also taken from [5, 6, 8]) with the following transfer function,

$$H(z) = \frac{1 + 0.1z^{-1} - 3.2725z^{-2} + 1.41125z^{-3}}{1 - 1.9z^{-1} + 1.1525z^{-2} - 0.1625z^{-3}}, \tag{18}$$

were used to generate the synthetic noisy data $x(n)$ of length $N = 2048$ for SNR = 27 dB and measurement noise $w(n)$ being white Gaussian.

Because $\gamma_3 = 0$ (skewness) and $\gamma_5 = 0$ but $\gamma_4 = 0.27$ (kurtosis), $\gamma_2 = \sigma_u^2 = \lambda\sigma_r^2 = 0.1$ and $\gamma_6 = 1.08$ for this case, the new inverse filter criterion $J_{2,6}$ was used to estimate the causal inverse filter $v(n)$ of order $L = 24$. The initial condition $\boldsymbol{v} = [0, \ldots, 0, 1, v_b(1), \ldots, v_b(12)]^T$, where $\{1, v_b(1), \ldots, v_b(12)\}$ were the coefficients of a twelfth-order LPE filter obtained by Burg's algorithm, was used to initialize the associated gradient type optimization algorithm for searching for the optimum $\hat{\boldsymbol{v}}$. The obtained inverse filter estimate (dash-dotted line) normalized by $\|\hat{\boldsymbol{v}}\| = 1$ is shown in Fig. 2 together with the true noncausal stable inverse filter $h_I(n)$ (solid line) also normalized by $\sum_{n=-\infty}^{\infty} |h_I(n)|^2 = 1$ where the unknown time delay between $\hat{v}(n)$ and $h_I(n)$ was artificially removed. For comparison, a conventional minimum-phase LPE filter $v_b(n)$ of order equal to 24 was also obtained by Burg's algorithm, which is depicted by a dashed line in Fig. 2. Note, from Fig. 2, that $\hat{v}(n)$ is quite close to the true inverse filter $h_I(n)$ but is very different from the LPE filter in waveshape. The data $x(n)$ were then processed by the LPE filter to obtain the predictive deconvolved signal $e_b(n)$ which is depicted by a dotted line in Fig. 3(a) for $N = 0$–511 together with the true input sequence $u(n)$ depicted by a solid line. One can observe, from Fig. 3(a), that in addition to a scale factor, each spike in $u(n)$ is associated with a residual wavelet

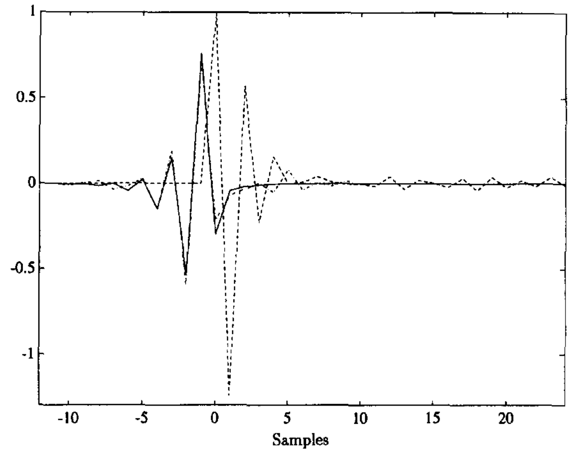
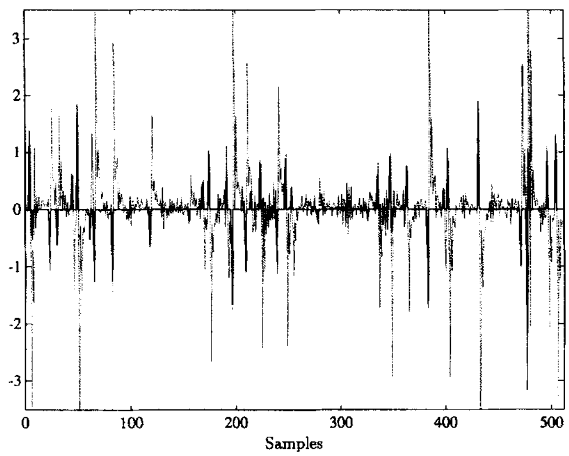


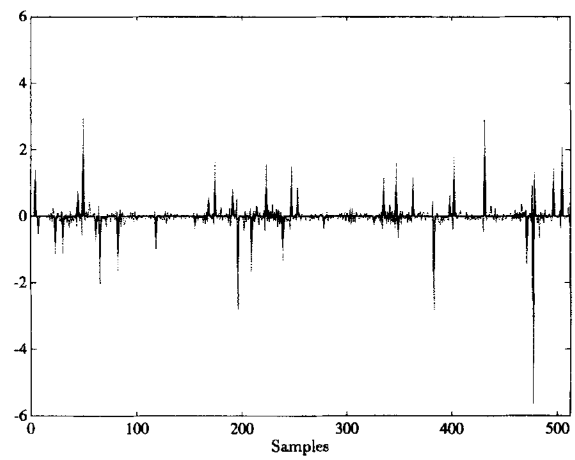
Fig. 2. Simulation results for Example 2. The true noncausal stable inverse filter $h_I(n)$ (solid line) normalized by $\sum_{n=-\infty}^{\infty} |h_I(n)|^2 = 1$, the inverse filter estimate (dash-dotted line) associated with $J_{2,6}$ where the unknown time delay was artificially removed, and the LPE filter (dashed line) of order equal to 24 obtained by Burg's algorithm.

which begins with two opposite peaks and gradually decays. The reason for this is simply that an all-pass distortion remains in $e_b(n)$ because only the amplitude response of nonminimum-phase source wavelet can be equalized by $v_b(n)$. The deconvolved signal $e(n)$ (dotted line) obtained by the optimum inverse filter depicted by a dash-dotted line in Fig. 2 is shown in Fig. 3(b) for $N = 0$–511 together with the true input sequence $u(n)$ (solid line). One can see, from Fig. 3(b), that $e(n)$ approximates $u(n)$ well except for a scale factor. Comparing the deconvolved signal shown in Fig. 3(a) with the one shown in Fig. 3(b), one can easily see that $e(n)$ is indeed a much better estimate of $u(n)$ than $e_b(n)$ because the phase distortion (all-pass distortion) in $e_b(n)$ (dotted line in Fig. 3(a)) was almost inexistent in $e(n)$ (dotted line in Fig. 3(b)). The deconvolved signals $e_b(n)$ and $e(n)$ for $N = 512$–2047 are omitted here since they have the same characteristics as those shown in Figs. 3(a) and 3(b), respectively. These simulation results support the fact that the proposed inverse filter criterion $J_{2,6}$ can be used for deconvolution.

As discussed in Example 1, one can surely use Tugnait's criterion $J_{2,4}$ rather than the new criterion $J_{2,6}$ for this example because $\gamma_4 = 0.27 \neq 0$.



(a)



(b)

Fig. 3. Simulation results for Example 2. (a) The predictive deconvolved signal $e_b(n)$ (dotted line) together with the true input signal (solid line) for $N = 0$–511; (b) the deconvolved signal $e(n)$ (dotted line) obtained by the optimum inverse filter associated with $J_{2,6}$ together with the true input signal (solid line) for $N = 0$–511.

This example only emphasizes the application of the new $J_{2,6}$ to deconvolution although both of $J_{2,4}$ and $J_{2,6}$ are members of the unified class of $J_{2s,l+s}$ where $l > s \geqslant 1$ presented in Theorem 1.

4. Conclusions

We have shown that the cumulant based inverse filter criteria $J_{r,m}$ given by (3) which use an mth-order cumulant and an rth-order cumulant for blind deconvolution and equalization require r to be even and $m > r$ (see Theorem 1). Therefore, these criteria form a family of criteria $J_{2s,l+s}$ where $l > s \geqslant 1$ and they include not only the existing inverse filter criteria as special cases of (l, s) but also new inverse filter criteria (see (R1)). The optimum inverse filter associated with $J_{2s,l+s}$ is a consistent estimate (see (R2)) and can be obtained by iterative nonlinear optimization algorithms which can only guarantee a local optimum solution. Some simulation results were provided to support that $J_{2s,l+s}$ with $l > s \geqslant 1$ is effective.

Appendix A

Proof of Lemma 1

Assume that

$$\max\{|a(n)|\} = \beta > 0. \tag{A.1}$$

Since $\max\{|a(n)|/\beta\} = 1$ and $l > s \geqslant 1$, one can easily infer that

$$1 \leqslant \sum_{n=-\infty}^{\infty} \left(\frac{|a(n)|}{\beta}\right)^l \leqslant \sum_{n=-\infty}^{\infty} \left(\frac{|a(n)|}{\beta}\right)^s, \tag{A.2}$$

which further leads to

$$\left\{\sum_{n=-\infty}^{\infty} \left(\frac{|a(n)|}{\beta}\right)^l\right\}^{1/l} \leqslant \left\{\sum_{n=-\infty}^{\infty} \left(\frac{|a(n)|}{\beta}\right)^s\right\}^{1/s}. \tag{A.3}$$

Cancelling the common term β on both sides of (A.3) yields

$$\left\{\sum_{n=-\infty}^{\infty} |a(n)|^l\right\}^{1/l} \leqslant \left\{\sum_{n=-\infty}^{\infty} |a(n)|^s\right\}^{1/s}. \tag{A.4}$$

□

Appendix B

Proof of Theorem 1

It can be easily shown [14, 15] that for either $m = 2$ with SNR $= \infty$ or $m \geqslant 3$, the mth-order cumulant of $e(n)$ is given by

$$C_m = \gamma_m \sum_{n=-\infty}^{\infty} g^m(n), \tag{B.1}$$

where $g(n)$ was defined by (6). Substituting (B.1) into (3) gives rise to

$$J_{r,m}(v(n)) = \frac{|\gamma_m|^r}{|\gamma_r|^m} \frac{|\sum g^m(n)|^r}{|\sum g^r(n)|^m}. \tag{B.2}$$

Let us consider $J_{r,m}$ given by (B.2) for the following three cases: (C1) $m < r$; (C2) $m > r$ and r is odd; and (C3) $m > r$ and r is even, respectively.

(C1) *$m < r$.* Let us give a counterexample to show that $J_{r,m}$ is unbounded for this case. Assume that

$$v(n) = \sum_{k=1}^{K} \alpha h_{\mathrm{I}}(n-k), \tag{B.3}$$

where α is a nonzero constant. Then

$$g(n) = h(n) * v(n) = \sum_{k=1}^{K} \alpha\delta(n-k) \tag{B.4}$$

since $h_{\mathrm{I}}(n)$ is the inverse filter of $h(n)$. Substituting (B.4) into (B.2) gives rise to

$$\begin{aligned} J_{r,m}(v(n)) &= \frac{|\gamma_m|^r}{|\gamma_r|^m} \frac{|\sum_{k=1}^{K} \alpha^m|^r}{|\sum_{k=1}^{K} \alpha^r|^m} \\ &= \frac{|\gamma_m|^r}{|\gamma_r|^m} \frac{|K\alpha^m|^r}{|K\alpha^r|^m} \\ &= \frac{|\gamma_m|^r}{|\gamma_r|^m} K^{(r-m)}. \end{aligned} \tag{B.5}$$

Note, from (B.5), that when K is increased, $J_{r,m}(v(n))$ becomes unbounded since $m < r$. Therefore, $J_{r,m}$ is unbounded for this case.

(C2) *$m > r$ and r is odd.* Let us also give a counterexample to show that $J_{r,m}$ is unbounded for this case. Assume that

$$v(n) = \alpha h_{\mathrm{I}}(n) + \alpha h_{\mathrm{I}}(n-1) - (2)^{1/r}\beta h_{\mathrm{I}}(n-2), \tag{B.6}$$

where $\alpha > 0$ and $\beta > 0$. Then

$$\begin{aligned} g(n) &= h(n) * v(n) \\ &= \alpha\delta(n) + \alpha\delta(n-1) - (2)^{1/r}\beta\delta(n-2). \end{aligned} \tag{B.7}$$

Substituting (B.7) into (B.2) yields

$$J_{r,m}(v(n)) = \frac{|\gamma_m|^r}{|\gamma_r|^m} \frac{|2\alpha^m + (-1)^m 2^{m/r}\beta^m|^r}{|2\alpha^r - 2\beta^r|^m}. \tag{B.8}$$

Note, from (B.8), that when α approaches β, the numerator of $J_{r,m}$ never approaches zero since $2^{m/r} > 2$ but the denominator of $J_{r,m}$ will approach zero. Therefore, $J_{r,m}$ is unbounded for this case when α approaches β.

(C3) *$m > r$ and r is even.* Alternatively, let $r = 2s < m = l + s$ where $l > s \geqslant 1$. Then (B.2) can be expressed as

$$\begin{aligned} J_{2s,l+s}(v(n)) &= \frac{|\gamma_{l+s}|^{2s}}{|\gamma_{2s}|^{l+s}} \frac{[\sum g^{l+s}(n)]^{2s}}{[\sum g^{2s}(n)]^{l+s}} \\ &= \frac{|\gamma_{l+s}|^{2s}}{|\gamma_{2s}|^{l+s}} \frac{[\sum g^{2s}(n) \sum g^{2l}(n)]^s}{[\sum g^{2s}(n)]^{l+s}} \frac{[\sum g^{l+s}(n)]^{2s}}{[\sum g^{2s}(n) \sum g^{2l}(n)]^s} \\ &= \frac{|\gamma_{l+s}|^{2s}}{|\gamma_{2s}|^{l+s}} \frac{[\sum g^{2l}(n)]^s}{[\sum g^{2s}(n)]^l} \left\{ \frac{[\sum g^{l+s}(n)]^2}{\sum g^{2s}(n) \sum g^{2l}(n)} \right\}^s. \end{aligned} \tag{B.9}$$

Note that the second term on the right-hand side of (B.9) is bounded by unity, i.e.,

$$\frac{[\sum g^{2l}(n)]^s}{[\sum g^{2s}(n)]^l} = \left\{ \frac{[\sum (g^2(n))^l]^{1/l}}{[\sum (g^2(n))^s]^{1/s}} \right\}^{ls} \leqslant 1$$

$$\text{(by Lemma 1).} \tag{B.10}$$

Moreover,

$$\left\{ \frac{[\sum g^{l+s}(n)]^2}{\sum g^{2s}(n) \sum g^{2l}(n)} \right\}^s \leqslant 1$$

$$\text{(by Cauchy–Schwarz inequality).} \tag{B.11}$$

Therefore,

$$\max\{J_{2s,l+s}(v(n))\} = \frac{|\gamma_{l+s}|^{2s}}{|\gamma_{2s}|^{l+s}}, \tag{B.12}$$

which occurs only when the equality simultaneously holds in both (B.10) and (B.11). Thus, we have completed the proof for statement (S1). Next, let us find the optimum inverse filter $\hat{v}(n)$ associated with $\max\{J_{2s,l+s}(v(n))\}$ given by (B.12).

The equality of (B.11) holds if and only if

$$g^s(n) = \beta g^l(n), \tag{B.13}$$

where β is an arbitrary nonzero constant. Without loss of generality, assume that $\beta > 0$. It can be easily inferred, from (B.13), that

$$g(n) = \begin{cases} \alpha q(n), & l-s \text{ is odd}, \\ \pm\alpha q(n), & l-s \text{ is even}, \end{cases} \tag{B.14}$$

where $\alpha = \beta^{1/(s-l)}$ and $q(n)$ is a binary sequence of

$\{0,1\}$. Substituting (B.14) into (B.10) gives rise to

$$\frac{[\sum g^{2l}(n)]^s}{[\sum g^{2s}(n)]^l} = \frac{1}{[\sum q(n)]^{l-s}} \leqslant 1 . \qquad \text{(B.15)}$$

Since $\sum q(n) \geqslant 1$ and $l > s$, the equality in (B.15) occurs when $\sum q(n) = 1$. In other words, $q(n) = \delta(n-\tau)$ or $g(n) = \pm\alpha\delta(n-\tau)$, where τ is an unknown integer. Thus, we have completed the proof for statement (S2). □

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